Chapter 1

MODELING COMPETITION AMONG WIRELESS SERVICE PROVIDERS

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Abstract We consider a scenario where a population of customers is spatially distributed in a region which is served by two wireless service providers that offer Internet Access via two noninterfering technologies: one having a uniform coverage over the region (e.g. WAN), and the other, a limited coverage (e.g. WiFi "hotspots"). We assume that customers are equipped with "dual mode" wireless communication devices that have the capability to select which among the available providers to use. We introduce a stochastic geometric model for the locations of customers and providers' access points and a utility-based mechanism modeling how devices select among providers. In particular, we assume that each device makes greedy decisions at random times, i.e., selects the available provider offering the highest utility at that time. We demonstrate that this process may have multiple equilibria, and prove that the system will almost surely evolve to one of the equilibrium configurations, starting from any initial configuration for users' choices. We also provide conditions under which the set of equilibria is relatively "tight" – in this case the equilibrium configuration of agents' choices is "maxmin fair" and thus is desirable if providers wish to cooperate in providing users with worst case performance guarantees. As an application of our framework we analyze the WAN and WiFi competition in an asymptotic scenario where the service zones of WAN provider are much larger than those of WiFi access providers.

Keywords: 3G, WiFi, multi-mode devices, decision making, heterogeneous wireless networks, multi-provider wireless networks, competition, equilibria, performance, scalability

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Introduction

Moving decision-making from access points to communication devices provides a path to achieving scalability in future complex and diverse networking landscapes [1]. Thus, we believe that increasingly, end-nodes will have the capability to use multiple communication modes to transfer data among themselves and/or connect to the wired network. For example, a “dual-mode” phone may be able to connect to a wide area cellular network or to an IEEE 802.11 LAN access point [2]. Users of such devices are able to decide which mode of communication they will use. In fact, such decision-making would likely be carried out by a software “agent” driven by users’ preferences or engineering design goals. For example, decisions could be based on proximity, amount of interference, quality of service, or, more abstractly, based on a utility function capturing a user’s valuation for the available services and their respective costs. Furthermore, decisions might be based on “local” estimates and/or “global” signaling from providers, e.g., a “price” signal. Through such signals, the providers can guide agent’s local decisions towards ones that are system or socially optimal.

Giving such freedom of choice to end-nodes is likely to affect system performance, and will result in competition among devices for the best resource (e.g. access point) as well as competition among providers to get a larger share of subscribers. This paper is a first step towards understanding such competition. We consider a scenario where a spatially distributed population of customers are equipped with dual mode devices and are served (on the downlink) by two wireless service providers. We assume that one of the providers utilizes a wide area network (WAN) technology, e.g. IS95, whereas the other provider uses a set of non-interfering wireless local area network access points (APs) e.g. IEEE 802.11 (WiFi “hotspots”). Our objective is to develop a framework to analyze the interplay among the agents’ decision rules, technological aspects, such as coverage and aggregate bandwidth available at the access points, and the densities of agents and access points, will affect the ability of providers to compete for a share of subscribers.

In Section 1 we formulate the stochastic geometric model for providers’ service zones and define utility-based decision rules. In Section 2 we prove convergence to equilibrium configurations for agents’ decisions, and then investigate the properties of the equilibrium in Section 3. Lastly, in Section 4 we will demonstrate how our results can be used to estimate the regimes where the hotspots and WAN APs are competitive, i.e. the majority of agents exert nontrivial choices.

1. Model for the network geometry and agents' decisions

To model the geometry of the network we use the stochastic-geometric framework introduced in [4]. The basic idea is to represent the locations of subscribers and access points as realizations of spatial point processes (e.g. Poisson) and the service zones associated with the access points as functionals of the realizations of these processes. The main advantage of such models is that they allow one to analytically capture the effect of spatial variations in the system based on a reduced set of salient parameters.

We will use three point processes Π^a, Π^h and Π^w, to represent the locations of agents, hotspots and WAN APs respectively. At this point we do not restrict ourselves by considering particular distributions those processes might have. Instead, we require all three of them be simple processes (see, e.g. [5]) so that the location of each WAN or hotspot AP is not shared by any other AP. Below we define some notation that will be used throughout the paper.

- $\pi^a = \{a_i\}_{i=1}^{\infty}$, $\pi^h = \{h_k\}_{k=1}^{\infty}$ and $\pi^w = \{w_m\}_{m=1}^{\infty}$ – represent realizations of Π^a, Π^h and Π^w on the plane. For brevity, we use a_i to denote both the agent and its location (similarly for hotspots and WAN APs).
- $\pi(A)$ – all points of the realization π of a point process Π that fall within the set A.
- $\left|\pi(A)\right|$ – the number of points of the realization π that fall within the set A.
- $|x|$ – the length of vector $x \in \mathbb{R}^2$.
- $B(x,r)$ – the disc of radius r centered at point $x \in \mathbb{R}^2$.
- V_m^w – the Voronoi cell of WAN AP $w_m \in \pi^w$. (The Voronoi cell associated with the point y_i of realization π is defined as the set of points on the plane that are closer to y_i than to any other point $y_j \in \pi \setminus \{y_i\}$.)
- V_k^h – the Voronoi cell of hotspot AP $h_k \in \pi^h$.
- $\mathcal{K}_m = \{k : \ h_k \in \pi^h(V_m^w)\}$ – the indices of hotspots located within the Voronoi cell V_m^w.
- S_k^h – the service zone (see below) of hotspot AP h_k.
- S_m^w – the service zone of WAN AP w_m.

We will refer to the "service zone" of WAN or hotspot AP as the set of locations on the plane, that the AP can serve. We assume that agents which fall within the service zones of several APs are able to choose which AP to connect to. In the next few paragraphs we describe our models for the service zones associated with each AP as well as the criterion each agent uses to connect to a particular access point.

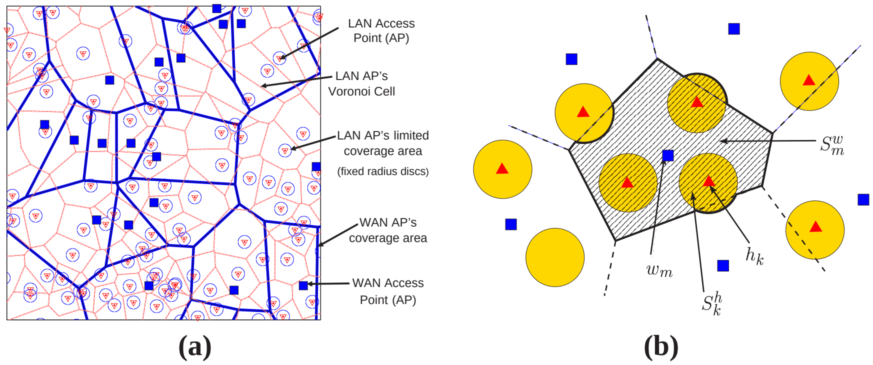

Figure 1.1. **(a)**: Boxes represent APs of the WAN provider, whereas triangles represent the APs of the hotspots. The coverage area of each box is modeled by its Voronoi cell, while that of the triangles is modeled by discs of radius d centered at the triangles. **(b)**: Voronoi cell of WAN AP "augmented" with hotspots' service zones as the service zone for this WAN AP.

Service zones for hotspots. Note that the coverage of a hotspot is usually limited due to constraints on transmit power of devices operating in unlicensed spectrum. Thus with each hotspot AP $h_k \in \pi^h$ we associate a disc $B(h_k, d)$ of some radius $d > 0$ and assume that the service from h_k is feasible only within this disc (see Figure 1.1 (a)). In addition, we assume that agents desiring to connect to a hotspot will connect only to the closest feasible hotspot, which yields a service zone S_k^h for hotspot AP h_k given by:

$$S_k^h \triangleq V_k^h \cap B(h_k, d)\,.$$

Service zones for the WAN. By contrast with hotspots, WAN service covers all spatial locations. Still, the definition of service zones depends on the underlying technology. For instance, in CDMA-based technologies the association of mobiles with APs is different for the up- and down-links [6]. Moreover, the service zones corresponding to two different WAN APs are in general not disjoint and, in fact, overlap to permit soft handoffs.

Appropriate models for CDMA service zones have been recently considered in [7], [8]. In particular the authors have shown that under some conditions (large enough power at APs, large attenuation) the service zone associated with any AP converges to its associated Voronoi cell. This suggests that Voronoi cells might be a reasonable model for service zones.

Note, however, that if we represent the service zones as Voronoi cells, agents that belong to hotspots that are crossed by the boundary of a Voronoi cell associated with some WAN AP might be choosing between this hotspot and one of two WAN APs. This poses certain problems in the analysis of the model, because agents' decisions interact across WAN AP service zones. To overcome this difficulty we shall impose a constraint that each agent $a_i \in \pi^a$ selects between the closest hotspot AP h_k (if it is covered by its service zone) and the WAN AP w_m *which contains* h_k in its service zone (see Figure 1.1 (b)). Whenever the hotspots' service range d is much smaller than the average size of a WAN cell, this assumption will not significantly affect our results. We will define the service zone of WAN AP w_m as the "augmented" Voronoi cell V_m^w:

$$S_m^w = V_m^w \bigcup \left(\bigcup_{k \in \mathcal{K}_m} S_k^h \right) \Bigg\backslash \left(\bigcup_{l \in \cup_{n \neq m} \mathcal{K}_n} S_l^h \right) .$$

Assumption 1.1. *The service zones S_m^w, $\forall m \in \mathbb{N}$, contain an a.s. finite number of agents and hotspots.*

Agents' selection criterion. Let C_m be the subset of S_m^w that includes only the area where agents have the option to choose among a hotspot and WAN AP w_m:

$$C_m \triangleq \bigcup_{k \in \mathcal{K}_m} S_k^h ,$$

and let $\overline{C}_m = S_m^w \setminus C_m$. We assume that any agents whose location is in $\overline{C}_m$ can not make a choice and thus connect to the WAN AP w_m. By contrast, an agent $a_i \in C_m$ is also covered by some hotspot h_k's service zone and can choose between *either* connecting to h_k or the WAN AP w_m.

Consider an agent a_i that is connected to a WAN AP at time t. We model her level of satisfaction with the service via a the utility function $U^w(N^w(a_i,t))$ of the total number of agents $N^w(a_i,t)$ that at time t are connected to the same WAN AP as agent a_i. Similarly, we assign a utility function $U^h(N^h(a_j,t))$ to an agent a_j to model her level of satisfaction if she is connected to a hotspot, where $N^h(a_j,t)$ denotes the total number of agents that are connected at time t to the same hotspot as the agent a_j. Thus, in this framework, utility functions are only "congestion" dependent and independent of positions of agents relative to the access points[1]. In the sequel we will use the following assumption for the utility functions:

Assumption 1.2. *$U^w(\cdot) : \mathbb{R}^+ \mapsto \mathbb{R}$ and $U^h(\cdot) : \mathbb{R}^+ \mapsto \mathbb{R}$ are continuous, monotonically decreasing functions.*

Now we describe how we model the decision process in this system. We postulate that an agent $a_i \in C_m$ switches at time t to the WAN AP w_m from its hotspot if and only if she was connected to this hotspot AP at time t^- and

$$U^w\Big(N^w(a_i,t^-)+1\Big) > U^h\Big(N^h(a_i,t^-)\Big) ,$$

where t^- refers to the time immediately prior to t. Similarly, the agent a_i switches to a hotspot AP at time t if and only if she was connected to a WAN AP at t^- and

$$U^h\left(N^h(a_i,t^-)+1\right) \geq U^w\left(N^w(a_i,t^-)\right).$$

Note that we break ties in favor of hotspots.

Assumption 1.3. *Agents' decision times within C_m are given by a simple point process Φ_m with realizations ϕ_m which obey the following:*

- *ϕ_m almost surely contains infinitely many points in $\mathbb{R}^+$, i.e. $\phi_m = \{s_k\}_{k=1}^{\infty}$, where $s_k \in \mathbb{R}^+$ for $k = 1,2,\dots$*
- *each point of ϕ_m is associated with a decision time for a unique agent within C_m*
- *a point $s_k \in \phi_m$ is a decision time of the agent $a_i \in C_m$ with some* **positive** *probability p_i, which possibly depends on realization ϕ_m up to time s_k and the history of agents choices up to time s_k.*

Assumption 1.3 postulates that only one agent within C_m can make decision at a time, each agent has unlimited opportunities for decision making, and any decision time with some positive probability is associated with a particular agent.

2. Convergence to equilibrium.

We call a particular configuration of agent's choices an equilibrium configuration, if and only if the system remains in this configuration indefinitely provided it starts in this configuration.

Proposition 2.1. *(**Convergence to equilibrium**.) Consider the service zone S_m^w for a particular fixed realization π^a, π^h and π^w. Then under Assumptions 1.1-1.3, given any initial configuration of connections at time $t = 0$, the system converges a.s. to an equilibrium configuration as $t \to \infty$.*

Here we will give the essential idea of the proof whereas the rest of the details we placed in Appendix 1.A.1. Note that the dynamics of the configuration of agents' decisions in S_m^w follow a continuous-time Markov chain with state $\mathcal{X}(t) := \{X(a_i,t)|\ a_i \in \pi^a(\mathcal{C}_m)\}$, where $X(a_i,t) \in \{0,1\}$ denotes the "connection state" of the agent a_i at time t. It takes the value 0 if the agent is connected to a hotspot, and 1 if she is connected to a WAN AP. We shall classify decision times for this chain as "up", "down" and "stay", corresponding to an agent switching from a hotspot to the WAN AP, vice versa, or staying with her current choice. For simplicity we uniformize the continuous time chain, and focus on a discrete-time Markov chain capturing the state at decision times.

We shall denote these times via $s = 1, 2, \ldots$. The transition probabilities for the discrete-time Markov chain are comprised of two factors: the probability that a particular agent reconsiders her decision at that time and whether, given the current configuration, the agent would change providers.

By Assumption 1.1, each service zone contains an a.s. finite number of agents, thus there is an a.s. finite number of possible configurations for agents' choices. It follows that some of the states must be revisited by the chain infinitely often. To show the convergence of the process to an equilibrium, it suffices to construct a feasible path for the chain evolution which starting from any initial configuration hits an equilibrium state, and has a positive probability of occurring. Since the state space is a.s. finite, and at least one state is visited infinitely often, this guarantees that the chain is transient, i.e. reaches an equilibrium state with probability 1.

Below we describe the steps of an algorithm to construct a path $\mathcal{P}$ consisting of a sequence of transitions for the state $\mathcal{X}(s)$, which, starting from any arbitrary configuration of agents' choices $\mathcal{X}(0)$, ends up in an equilibrium configuration after a finite number of steps. Let $A^u(s)$ denote the set of agents that, given the configuration at time s, could make "up" transitions and $A^d(s)$ the set of agents that can make "down" transitions. We describe our algorithm in terms of the pseudo-code shown in Table 1.1. Note that the algorithm assumes that an agent making her decision at time slot $s \geq 1$ is basing this decision by observing the state of the system prior to that time, i.e. at time $s-1$.

In short, after initialization, the algorithm alternates between phases where "up" and "down" transitions occur. During the Up-transition phase only the "up"-switches occur, and agents performing these transitions are selected from the most "congested" hotspots. This phase ends once the set of agents able to perform the "up" switches is empty. At that time the algorithm switches to the Down-transition phase, where at most one agent performs a "down" switch. If an agent performs an "up" switch at time s, we track the number of agents, $K(s)$, that shared the hotspot with this agent prior to her switching at time s.

To show that this algorithm finishes in finite time in Appendix 1.A.1 we prove that $K(s), s = 1, 2, \ldots$ is a non-increasing sequence that at each time bounds above the number of agents within each hotspot in S^w_m. Thus since $K(\cdot)$ is integer valued and non-negative, it must converge to some value K^*_m in an a.s. finite time. Once $K(\cdot)$ converges, we prove that only down transitions can occur, and since there is an a.s. finite number of agents in each WAN APs service location, an equilibrium must be reached in finite time.

In summary, we have shown that from any starting configuration there exists a path, $\mathcal{P}$, that reaches an equilibrium state. Moreover, by Assumption 1.3 the overall probability of the path $\mathcal{P}$ is strictly positive. Since the state space is finite, there must be a state which is visited infinitely often. Whence the Markov chain will necessarily eventually hit an equilibrium state.

Initialization:
$s = 1$ and $\mathcal{X}(s) = \mathcal{X}(0)$
go to Up-transition phase

Up-transition phase:
repeat:
{ choose $a_j = \arg\max_{a_i \in A^u(s)} N^h(a_i, s)$
$K(s) := N^h(a_j, s)$
let a_j make an "up" transition
update the state $\mathcal{X}(s)$
$s := s + 1$ }
until $A^u(s) = \emptyset$
go to Down-transition phase

Down-transition phase:
if $A^d(s) \neq \emptyset$:
{ choose any $a_j \in A^d(s)$
let a_j make a "down" transition
update the state $\mathcal{X}(s)$
$K(s) := K(s-1)$
$s := s + 1$
go to Up-transition phase }
otherwise: done

Table 1.1. Pseudo-code for constructing the path $\mathcal{P}$ converging to equilibrium.

3. Structure of equilibrium.

In this section we will give a characterization of the system state, i.e., configuration of agents' decisions, define a notion of uniqueness, and analyze under what conditions the system equilibrium is unique. We first introduce some additional notation[2]

- $M^w_m = \left|\pi^a(S^w_m)\right|$ – the number of agents located within the service zone of WAN AP w_m.
- $M^h_k = \left|\pi^a(S^h_k)\right|$ – the number of agents located within the service zone of hotspot h_k.
- $M^w_{C_m} = \left|\pi^a(C_m)\right|$ – the number of agents located within C_m, i.e. agents that can make choices.
- $M^w_{\overline{C}_m} = M^w_m - M^w_{C_m}$ – the number of agents located within $\overline{C}_m$, i.e. the agents that can *not* make choices.
- $H_m = \left|\pi^h(S^w_m)\right|$ – the number of hotspots located within the service zone of WAN AP w_m.
- $\mathcal{X}^{\pi^a,\pi^h,\pi^w}_m \triangleq \{X(a_i)|\ a_i \in \pi^a(C_m)\}$ – denotes the system configuration in service zone S^w_m associated with a fixed realization π^a, π^h and π^w. Here $X(a_i) \in \{0,1\}$ takes the value 0 if agent a_i is connected to a hotspot, and 1 if she is connected to a WAN AP.
- $\mathcal{T}_m = \mathcal{T}_m(\pi^a,\pi^h,\pi^w)$ – the a.s. finite set of possible system configurations (states) in S^w_m for a given realization π^a, π^h and π^w.
- $\mathcal{E}_m$ – the set of all system configurations $c \in \mathcal{T}_m$ that correspond to equilibria in S^w_m.
- $\mathcal{F}_m = \mathcal{F}_m(\pi^a,\pi^h,\pi^w)$ – subset of $\mathcal{E}_m$ which consists of only "fair" equilibria (see below).
- $N^w_m(c)$ – the number of agents that connect to WAN AP w_m in configuration $c \in \mathcal{T}_m$.
- $N^h_k(c)$ – the number of agents that connect to hotspot AP h_k in configuration $c \in \mathcal{T}_m$.
- $(U^w)^{-1}(\cdot)$ – unique and decreasing, by Assumption 1.2 inverse of $U^w(\cdot)$.
- $(U^h)^{-1}(\cdot)$ – unique and decreasing inverse of $U^h(\cdot)$.
- $G(\cdot) \triangleq (U^h)^{-1} \circ U^w(\cdot)$ – nondecreasing composition of $(U^h)^{-1}$ and $U^w(\cdot)$
- $J(\cdot) \triangleq (U^w)^{-1} \circ U^h(\cdot)$ – nondecreasing composition of $(U^w)^{-1}$ and $U^h(\cdot)$

Characterization of a configuration. For any fixed realization π^a, π^h and π^w consider only WAN APs w_m that have at least one hotspot in their service areas, i.e. $\mathcal{K}_m \neq \emptyset$. For such m we characterize the system configuration $c \in \mathcal{T}_m$ for the service zone S_m^w by a vector $\mathbf{N}_m(c) \triangleq \{N_k^h(c)|\ k \in \mathcal{K}_m\}$. The vector $\mathbf{N}_m(c)$ determines how many agents are connected to each hotspot h_k for $k \in \mathcal{K}_m$ in configuration $c \in \mathcal{T}_m$.

Definition 3.1. *We say that two configurations for agents' choices characterized by $\mathbf{N}_m(c)$ and $\mathbf{N}_m(c')$ are equivalent, and write $\mathbf{N}_m(c) \sim \mathbf{N}_m(c')$, if the components of the vector $\mathbf{N}_m(c)$ are a permutation of those of $\mathbf{N}_m(c')$.*

Fair equilibria.

Definition 3.2. *We say that a configuration $c \in \mathcal{T}_m$ is "fair" if its characterization $\mathbf{N}_m(c) = \{N_k^h(c)|k \in \mathcal{K}_m\}$ satisfies, for some $K \in \mathbb{Z}^+$:*

$$\forall k \in \mathcal{K}_m : \begin{cases} K-1 \leq N_k^h(c) \leq K\,, & \text{if } M_k^h \geq K\,, \\ N_k^h(c) = M_k^h\,, & \text{otherwise}\,. \end{cases}$$

If c is also an equilibrium configuration we say that c is a "fair" equilibrium.

We shall interpret this definition via Figure 1.2. The hexagonal region is a schematic representation of the service zone S_m^w, while the positions of the cylinders represent the locations of hotspots. The height of each cylinder represents the overall number of agents that fall within the service zone of a particular hotspot.

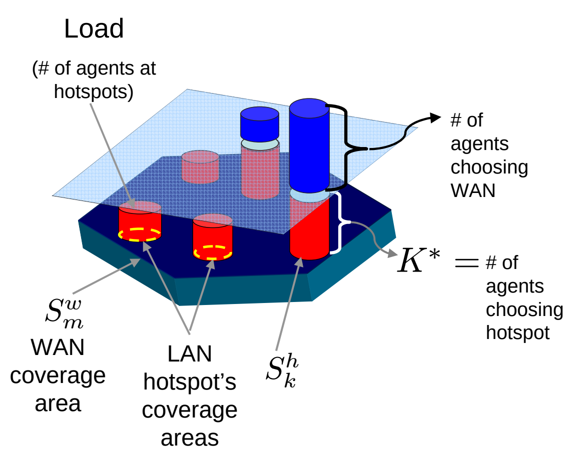

Figure 1.2. Structure of a "fair" configuration.

Assume that the slicing plane in Figure 1.2 is one unit thick and its upper surface is placed at integer-valued heights K above the surface of S_m^w. Any "fair" configuration has the following assignment of agents to APs:

- All agents in $S_m^w \setminus C_m$ connect to WAN AP w_m.
- A number of agents corresponding to the parts of cylinders that fall under the lower surface of the slicing plane connect to their respective hotspots.
- A number of agents corresponding to the parts of cylinders above the upper surface of the plane connect to the WAN AP w_m.
- Finally, a number of agents corresponding to the parts of cylinders within the slice connect to either their associated hotspots or WAN AP w_m.

In what follows, to avoid ambiguity we will always associate a fair configuration f with the "cutoff" plane at level[3] $K_m(f) = \max_{k \in \mathcal{K}_m} N_k^h(f)$. Note, that in fair configuration f the hotspots having more than $K_m(f)$ agents in their service zones yield the "overload" to the WAN AP w_m. As a result the number of agents connected to those hotspots is nearly the same, i.e. either $K_m(f)$ or $K_m(f) - 1$.

By the construction used to prove Proposition 2.1 we can always find a position of the slicing plane, $K = K_m^*$, and an assignment of agents corresponding to the parts of cylinders at the slice, so that the connection configuration in S_m^w is a fair equilibrium. This results in statement **(i)** of Proposition 3.1.

Proposition 3.1. *For any realization π^a, π^h and π^w we have that:*
(i) The set of all fair equilibria, $\mathcal{F}_m$, is not empty.
(ii) All fair equilibria have equivalent characterizations, i.e. for all $f, f' \in \mathcal{F}_m$, $\mathbf{N}_m(f) \sim \mathbf{N}_m(f')$.

For the proof of statement (ii) of Proposition 3.1, see Appendix 1.A.2.

Non-uniqueness of equilibrium.

Definition 3.3. *For a particular realization π^a, π^h and π^w we say that the equilibrium in S_m^w is unique if for any $e, e' \in \mathcal{E}_m$ we have $\mathbf{N}_m(e) \sim \mathbf{N}_m(e')$.*

Note that agents' decisions are discrete in nature, and unfortunately, this can lead to multiple equilibria in the system, even when we understand uniqueness in the weak sense of Definition 3.3. Below we show this via a simple example. Observe that for all equilibrim configurations $e \in \mathcal{E}_m$ we must have that:

$$U^h\left(N_k^h(e)+1\right) < U^w\left(N_m^w(e)\right) \text{ and } U^h\left(N_k^h(e)\right) \geq U^w\left(N_m^w(e)+1\right) \quad (1.1)$$

for all $k \in \mathcal{K}_m$ such that the service zone S_k^h has an agent connected to the WAN AP w_m and an agent connected to h_k. Also we must have that:

$$U^h\left(N_l^h(e)\right) \geq U^w\left(N_m^w(e)+1\right),$$

for all $l \in \mathcal{K}_m$ such that the service zone S_l^h has *all* of its agents connected to h_l. Lastly,

$$U^h(1) < U^w\left(N_m^w(e)\right), \tag{1.2}$$

must be satisfied for all $p \in \mathcal{K}_m$ such that all agents within S_p^h are connected to w_m in equilibrium. It follows from (1.1) that:

$$G\left(N_m^w(e)\right) - 1 < N_k^h(e) \leq G\left(N_m^w(e) + 1\right), \tag{1.3}$$

for hotspots $h_k \in \pi^h(S_m^w)$ with at least one agent connected to the WAN AP w_k. Note that, depending on the utility functions there can be more than one integer solution to the inequalities (1.3). Consider, for example:

$$U^h(N) = N^{-\beta}, \quad U^w(N) = N^{-\alpha}, \tag{1.4}$$

where $\alpha > \beta > 0$. In this case $G(N) = N^{\alpha/\beta}$, and the gap between the right and left hand side in (1.3) increases with $N_m^w(e)$. In other words, when the number of agents, not covered by hotspots is large enough, there can be many integer solutions to the inequalities (1.3). Hence, "unfair" equilibria can be constructed easily from the fair one. For example we could switch some number L of agents from WAN AP w_m to a particular hotspot h_k and the same number L of agents from some other hotspot h_l within the same WAN AP w_m. Note that this procedure would not change the number of agents connected to the WAN AP. If L is selected so that the $N_k^h(e) - L$ and $N_l^h(e) + L$ are still within the bounds (1.3), this procedure would result in a feasible equilibrium which is not equivalent to the fair one.

Conditions guaranteeing uniqueness and fairness. One might ask under what conditions the equilibrium in S_m^w is unique. The following result assumes that the utilities have a particular property, and that the cells of the WAN provider are large enough to guarantee that a sufficiently large number of agents connects to the WAN AP in equilibrium.

Proposition 3.2. *Suppose that there exists $\overline{N}$ such that for all $N \geq \overline{N}$*

$$G(N+1) - G(N) < 1, \tag{1.5}$$

and assume that the number of agents that can not make choices in service zone S_m^w satisfies:

$$M_{C_m}^w \geq \overline{N}. \tag{1.6}$$

Then, the equilibrium in S_m^w is unique and fair.

We prove this proposition in Appendix 1.A.3. In general, if the property (1.5) holds then it must be the case that the utility function associated with connections to hotspots decrements faster in the number of connected agents than the utility associated with connections to the WAN AP[4]. One such example is given by (1.4) with $\beta > \alpha > 0$.

System performance in equilibrium. Let us define $U_m^{min}(c)$ to be the minimum over the utilities of agents within S_m^w that choose according to configuration $c \in \mathcal{T}_m$. We refer to $U_m^{min}(c)$ as the utility of the bottleneck agent for the configuration c.

Proposition 3.3. *If the equilibrium in S_m^w is unique, then $U_m^{min}(c) \leq U_m^{min}(f)$, for all $f \in \mathcal{F}_m$ and $c \in \mathcal{T}_m$.*

We prove this proposition in Appendix 1.A.4. Thus, when equilibrium configuration of agents' choices is unique, it would maximize the utility of the bottleneck agents over all possible configurations of agents' choices. When agents' utilities are associated with a congestion-only-dependent performance metric, utility based choice mechanism would realize equilibria that are favorable from the point of view of worst case performance. We further explore the performance aspect of a multi-provider scenario in [11].

4. Estimation of competitiveness of WAN vs WiFi hotspots.

In this section we discuss how to compute the fractions of agents that are connected to WAN APs and hotspots in equilibrium. We choose these fractions to be our metric to assess the competitiveness of one provider versus another. For simplicity of exposition we assume[5] that $\Pi^{w,\alpha}$ is a deterministic process such that the Voronoi cells associated with each WAN AP are geometrically similar and have the same area α. We further assume that the processes Π^h and Π^a are stationary Poisson processes with densities λ^h and λ^a respectively.

The non-uniqueness of equilibria poses certain difficulties in analyzing the model for arbitrary utilities, densities and cell sizes. Note that in practice, the sizes of WAN service zones typically would exceed that of hotspots[6]. Thus, to simplify our analysis we will study the system where the size of WAN service zones, α is large enough to contain a large number of agents and hotspots. Intuitively, one might expect that when the WAN service zones grow in area, the set of different equilibria becomes tighter, i.e., a type of the Law of Large Numbers making the system more amenable to analysis. In the next paragraph we demonstrate that this intuition is indeed correct.

Setup for asymptotic analysis.. We consider a collection of deterministic point processes $\{\Pi^{w,\alpha}\}$ indexed by $\alpha \in \mathbb{R}^+$ where each represents the spatial locations of WAN APs that are increasingly spread out. In particular, we suppose that the area of the Voronoi cell associated with any point $w_m^\alpha \in \pi_\alpha^w$ is equal to α, and let α grow. Let us also assume that for each $\alpha > 0$, $\pi^{w,\alpha}$ contains a point w_0^α at the origin.

In what follows we will consider the service zones of WAN AP w_0^α and we will use the same notation as before to refer to the number of agents and hotspots falling within the service zone of the WAN AP $w_0^\alpha \in \pi^{w,\alpha}$, but indicate the dependence on the area α via the corresponding superscript. Thus, for example we will write H_0^α to indicate the number of hotspots that fall within the service zone $S_0^{w,\alpha}$ of the WAN AP w_0^α. In addition, we use $\mathbb{E}_0^h[A_0]$ to denote the expectation of the quantity A_k associated with a typical hotspot h_k (see, e.g. [10]).

For fixed λ^h and λ^a the service area of each WAN AP will have to support a larger (roughly linear in α) number of users as α grows. Therefore, we will assume that the WAN resources also scale with α. This leads to a scaling requirement on the utility function associated with connecting to the WAN. Let $U^{w,\alpha}(\cdot)$ denote the utility function associated with connecting to the WAN when the area of a Voronoi cell of any WAN AP is α, and assume that $U^{w,\alpha}(\cdot)$ satisfies Assumption 1.2 for utility functions. Define $J^\alpha(N) = (U^{w,\alpha})^{-1} \circ U^h(\cdot)$ (where $U^h(\cdot)$ is independent of α) and assume the following:

Assumption 4.1. *The scaling of $J^\alpha(N)$ with α is such that:*

1. $J^\alpha(N) = \alpha j(N)$ *for any* $N \in \mathbb{N}$,
2. $\lim_{N\to\infty} j(N) = \infty$,
3. *There exists* $\bar{N}$, *such that* $j^{-1}\Big((N+1)/\alpha\Big) - j^{-1}\Big(N/\alpha\Big) < 1$, *for all* $N \geq \bar{N}$ *and each* $\alpha > 0$.
4. *For any integer* $K \geq 2$, $u(K) \neq j(K), j(K-1)$, *where*

$$u(K) = \lambda^a e^{-\lambda^h \pi d^2} + \lambda^h \mathbb{E}_0^h \left[(M_0^h - K + 1) \mathbf{1}_{\{M_0^h \geq K\}} \right]. \tag{1.7}$$

The interpretation of these assumptions are as follows. Condition 1 means that the resources of WAN APs scale linearly in the area α of their service zones. For example, we might have $U^h(N) = \frac{B^h}{N}$ and $U^{w,\alpha}(N) = \frac{\alpha B^w}{N}$, in which case $J^\alpha(N) = \frac{\alpha B^w}{B^h} N$. The second condition follows if, as more agents connect to a resource, the utility of those agents is strictly decreasing to zero. The third condition will allow us to use Proposition 3.2 to argue that the equilibrium in $S_m^{w,\alpha}$ is unique with probability approaching 1 as $\alpha \to \infty$. Finally, the last condition is technical (see Appendix 1.A.5), and satisfied for the cases of interest.

We study the asymptotics of this system in Appendix 1.A.5. The results of our study are summarized in Theorem 4.1. Here when we say that an event E^α happens with high probability (w.h.p.) we mean that $\lim_{\alpha\to\infty} \mathbb{P}(E^\alpha) = 1$.

Theorem 4.1. *Consider any realization of the Poisson point processes* Π^a *and* Π^h *and the sequence of deterministic processes* $\{\Pi^{w,\alpha}\}$ *with Voronoi cells of area* α *and each with a typical cell centered at the origin. Under the scaling Assumption 4.1 we have:*

1 *The equilibrium* f_0^α *in* $S_0^{w,\alpha}$ *is unique and fair w.h.p.*

2 *The largest number of agents connected to each hotspot in this equilibrium,* $N_{\max}^{h,\alpha}(f_0^\alpha) = \max_{k\in\mathcal{K}_0^\alpha} N_k^h(f_0^\alpha)$ *has a limit:*

$$\lim_{\alpha\to\infty} N_{\max}^{h,\alpha}(f_0^\alpha) = N_{max}^{h,\infty},$$

for some integer $N_{max}^{h,\infty} \geq 0$.

3 *We have that* $N_{\max}^{h,\infty} > 0$ *if and only if* $j(1) \leq \lambda^a$ *in which case it is given by the largest integer solution for* $K \geq 1$ *of the inequality*

$$u(K) \geq j(K), \tag{1.8}$$

where $u(K)$ *is given by (1.7).*

The basic idea of the proof is to leverage the analogs of the Law of Large Numbers for functionals on random sets, e.g. Voronoi cells, which have distributions dependent on realizations of point processes. We also show that fluctuations from averages for the quantities of interest do not grow "too fast" as the area of the WAN service zones grows. This allows us to express the position of the asymptotic "cutoff" $N_{max}^{h,\infty}$, in terms of averages of functionals of the realizations of Π^h and Π^a.

Based on Theorem 4.1 the analysis of competition when the WAN cell sizes are "large" reduces to comparing the number $N_{max}^{h,\infty} = K^*$ to the average number of agents falling within the service zone of a typical hotspot. In particular, if

$$K^* \gg \mathbb{E}_0^h\left[M_0^h\right] = \frac{\lambda^a(1-e^{-\lambda^h\pi d^2})}{\lambda^h}, \tag{1.9}$$

then hotspots retain most of the agents that fall within their service zones in equilibrium. We classify this case as hotspots effectively competing with the WAN. On the other hand if

$$K^* \ll \frac{\lambda^a(1-e^{-\lambda^h\pi d^2})}{\lambda^h}, \tag{1.10}$$

the hotspots yield most of their agents to the WAN APs in equilibrium. In this case we say that hotspots are not competitive with respect to the WAN. Using

Theorem 4.1, we can suggest the following heuristic approach to estimate the value of N^h_{max}. In general one has to solve for $K \geq 0$ the equation:

$$U^w\left(\lambda^a|V|e^{-\lambda^h\pi|d|^2} + \lambda^h\mathbb{E}^h_0\left[P_0(K)\right]\right) = U^h(K)\,, \qquad (1.11)$$

where $P_0(K) = (M^h_0 - K + 1)\mathbf{1}_{\{M^h_0 \geq K\}}$. Note that since the left side of (1.11) is monotonically increasing in K and the right – monotonically decreasing, the solution either does not exist ($K^* = 0$) or is unique, when it exists. Unfortunately, there is no closed form expression for the term $\mathbb{E}^h_0[P_0(K)]$ and hence simulation has to be used to estimate it. However, to test if hotspots are not competitive with respect to the WAN one could use the following simple criterion. Clearly, (1.10) holds if the solution to:

$$U^w(\lambda^a|V| - \lambda^h K|V|) = U^h(K)\,, \qquad (1.12)$$

falls much below the value $\lambda^a/\lambda^h(1 - e^{-\lambda^h\pi d^2})$. Note that this allows for a simple intuitive interpretation. The number of agents and hotspots occupying WAN service zone tends to $\lambda^a|V|$ and $\lambda^h|V|$ respectively when $|V|$ is large. The number of agents connected in equilibrium to hotspots tends to $\lambda^h|V|K$, whenever $K \ll \lambda^a\mathbb{E}|S^h_k|$, since then we can assume that each hotspot has exactly K agents connected to its AP in equilibrium. Thus the number of agents connected to the WAN AP must tend to:

$$\lambda^a|V| - \lambda^h K|V|\,,$$

once the size of the WAN service zone gets large enough. Thus, (1.12) follows by equating the utility of agents that are connected to the WAN AP and utility of the ones that are connected to hotspots.

5. Conclusion

To summarize, we have developed a stochastic geometric model for a system where subscribers with dual mode devices select among two noninterfering wireless service providers – a WAN provider and a second provider (or aggregator) of LAN hotspots. Our model is of interest in that, on the one hand, it captures wireless providers using technologies that might have different capacity and coverage, and on the other hand it captures the role of subscribers decision-making mechanisms in determining the eventual equilibrium. Assuming each subscriber's decision-making agent makes greedy decisions, based on comparing two "congestion" dependent utilities, at random times, we show that an equilibrium configuration would eventually be reached. Further we have characterized such equilibria and shown that they are likely to be close to the fair equilibrium, which corresponds to slicing the excess loads on hotspots, and shifting these to the WAN. In an effort to get numerical estimates for the level at which this slicing occurs, we developed an asymptotic result for the case where WAN service areas are large, which would permit an evaluation of this setting.

The results in this paper can be viewed from different perspectives. On the one hand they permit an evaluation of the competitiveness of the two providers to attract subscribers in their service areas. On the other, they permit a study of how to design decision making mechanisms, i.e., appropriate utility functions, to realize equilibria that may be desirable equilibrium for the overall system. The highlight of this paper is a characterization of such equilibria, that would permit further consideration of the performance and network design implications of wireless systems where users are capable to switch among multiple providers, depending on the key parameters of the system.

Appendix

1. Details of proof of Proposition 2.1

Proof. Here we show that $K(s)$ is a non-increasing sequence. Indeed, during the execution of an Up-transition Phase, $K(s)$ may change, but can only be reduced, since only agents in $A^u(s)$, and which belong to the most congested hotspots, are selected to make a transition. Now suppose that an Up-transition phase finished at time τ, then $K(\tau-1)$ is the number of agents that shared the hotspot with the last eligible agent prior to her "up" transition. Consider $a_j \in A^d(\tau)$, an eligible agent for a down transition. Note that for any such agent it must be the case that

$$N^h(a_j,\tau) \le K(\tau-1)-2 \qquad (1.A.1)$$

otherwise the agent that switched up at time $\tau-1$ could not have improved her utility. Indeed, suppose at time $\tau-1$, the agent a_i switched "up", then the following inequality must have been true:

$$U^w\left(N^w(a_i,\tau-1)+1\right) > U^h\left(N^h(a_i,\tau-1)\right). \qquad (1.A.2)$$

Note that $N^w(a_j,\tau) = N^w(a_i,\tau-1)+1$, since both a_j and a_i belong to the same WAN service zone and no other transitions have occurred in the interim. Thus if

$$U^w\left(N^w\left(a_j,\tau\right)\right) \le U^h\left(N^h\left(a_j,\tau\right)+1\right)$$

this would contradict to (1.A.2) unless $N^h(a_j,\tau) \le N^h(a_i,\tau-1)-2 = K(\tau-1)-2$. Thus an agent that makes a "down" transition right after an Up-transition phase can not increase the number of agents on her hotspot beyond $K(\tau-1)-1$. Whence upon reentering the Up-transition phase, if up switches occur they can again only decrease the value of $K(\cdot)$.

Note, that if one or more "down" switches occur in sequence without any intermediate "up" transitions, it still remains the case that $K(s)$ must be an upper bound on the number of agents sharing a hotspot, of an agent that chooses to make an "up" transition at time s. Indeed, assume that the last Up-transition phase, that had an "up" switch, has finished at time $\tau-1<s$ and the agent a_i has switched "down" at time τ. The agent's a_i's switch has occurred due to the fact that:

$$U^w\left(N^w(a_i,\tau)\right) \le U^h\left(N^h(a_i,\tau)+1\right)$$

Note that for an agent a_j switching "down" at time $\tau+1$, we have $N^h(a_j,\tau+1) \geq N^h(a_j,\tau)$ and $N^w(a_j,\tau+1) = N^w(a_i,\tau) - 1$. Hence,

$$U^w\Big(N^w(a_j,\tau+1)\Big) \leq U^h\Big(N^h(a_j,\tau+1)+1\Big)$$

could only be feasible if $N^h(a_j,\tau+1) \leq N^h(a_i,\tau)$, by monotonicity of utilities. But then, in view of (1.A.1):

$$N^h(a_j,\tau+1)+1 \leq K(\tau)-1\,.$$

By induction, we can show that if $m+1$ such "down" transitions took place without any intermediate "up" transitions, then:

$$N^h(a_k,\tau+m)+1 \leq K(\tau)-1\,,$$

where a_k is the agent that has performed the last "down"-transition.

Thus $K(s)$ is a non-increasing sequence which bounds the number of agents connected to any hotspot at time s. Also since $K(\cdot)$ is integer valued sequence, it must converge to some value K_m^* in a finite time. Once $K(\cdot)$ converges, only down transitions can occur, and since there is an a.s. finite number of agents in each WAN APs service location, an equilibrium must be reached in finite time. □

2. Proof of Proposition 3.1

Proof. Consider any fair equilibrium configuration $f \in \mathcal{F}_m$ and let $K_m(f) = \max_{k\in\mathcal{K}} N_k^h(f)$ give the level of the corresponding slicing plane (see Figure 1.2). We will first show that for any two fair equilibria f and f' we have that $K_m(f) = K_m(f')$.

We show this by contradiction, suppose, in fact that there exist $f, f' \in \mathcal{F}_m$ such that $K_m(f) \neq K_m(f')$. Without loss of generality assume that $K_m(f) > K_m(f')$. Note that in this case for some $l \in \mathcal{K}_m$ we have $N_l^h(f) = K_m(f) \geq 1$. Considering the hotspot h_l, we get

$$U^h\Big(K_m(f)\Big) \geq U^w(N_m^w(f)+1) \tag{1.A.3}$$

since otherwise an agent connected to this hotspot would choose to switch to WAN AP w_m which would contradict the fact that f is an equilibrium. Now, for equilibrium f' all hotspots have fewer than or equal to $K_m(f') \leq K_m(f)-1$ agents, so in particular $N_l^h(f') \leq K_m(f)-1$. It follows by adding 1 to both sides and the fact that $U^h()$ is monotonically decreasing that:

$$U^h(N_l^h(f')+1) \geq U^h\Big(K_m(f)\Big). \tag{1.A.4}$$

At the same time, since $K_m(f') < K_m(f)$ it follows that $N_m^w(f') \geq N_m^w(f)+1$. Using the fact that $U^w()$ is monotonically decreasing we have that

$$U^w(N_m^w(f)+1) \geq U^w(N_m^w(f')). \tag{1.A.5}$$

Now putting (1.A.3),(1.A.4) and (1.A.5) together we have that

$$U^h(N_l^h(f')+1) \geq U^w(N_m^w(f'))$$

which implies that under f' an agent on WAN AP w_m would choose to switch to hotspot h_l. This contradicts the fact that f' is an equilibrium. Thus we conclude that for any $f \in \mathcal{F}_m$ we have $K_m(f) = K_m^*$ for some integer K_m^*.

In order to show that all fair equilibria are equivalent, we first argue that for two fair equilibria $f' \neq f$ we must have $N_m^w(f) = N_m^w(f')$. Without loss of generality suppose $N_m^w(f') \geq N_m^w(f)+1$. Then, for at least one hotspot, say h_l, $N_l^h(f') \leq N_l^h(f)-1$ which also implies that $N_l^h(f) \geq 1$. For f to be an equilibrium we must have that:

$$U^h\left(N_l^h(f)\right) \geq U^w(N_m^w(f)+1) \geq U^w(N_m^w(f')), \tag{1.A.6}$$

which follows from the fact that no agent in hotspot h_l wishes to switch to the WAN AP and our assumption. Considering the hotspot h_l under the equilibrium configuration f' we obtain:

$$U^w(N_m^w(f')) > U^h\left(N_l^h(f')+1\right) > U^h(N_l^h(f)), \tag{1.A.7}$$

which is the consequence of the fact that an agent in h_l connected to the WAN AP w_m has no desire to switch to the hotspot h_l. Clearly, by monotonicity of utilities we have that (1.A.6) is in contradiction to (1.A.7).

Thus we know that if $f, f' \in \mathcal{F}_m$, then we have $N_m^w(f) = N_m^w(f')$ and $K_m(f) = K_m(f') = K_m^*$, for some integer K_m^*. Next we show that all fair equilibria must have equivalent characterizations. Let R denote the number of hotspots in S_m^w that have at least K_m^*-1 agents in their service zones. The equilibrium number of agents connected to such hotspots is between K_m^*-1 and K_m^*. Now assume that $r < R$ of the R hotspots have K_m^*-1 agents and the remaining $R-r$ hotspots have K_m^* agents, connected to their APs under the equilibrium configuration f. Similarly, we assume that $r' < R$ hotspots have K_m^*-1 agents in the equilibrium configuration f'. Equating the total number of agents in the service zone S_m^w in equilibria f and f', we have that:

$$\begin{aligned}(K-1)r + K(R-r) + &\sum_{k\in\mathcal{K}_m,\, M_k^h<K_m^*-1} M_k^h + N_m^w(f) \\ &= (K-1)r' + K(R-r') + \sum_{k\in\mathcal{K}_m,\, M_k^h<K_m^*-1} M_k^h + N_m^w(f').\end{aligned}$$

Since $N_m^w(f) = N_m^w(f')$ this leads to $r = r'$, showing that $\mathbf{N}_m(f) \sim \mathbf{N}_m(f')$. □

3. Proof of Proposition 3.2

Proof. By part (i) of Proposition 3.1 there exists a fair equilibrium in S_m^w. Let $f \in \mathcal{F}_m$ be one such equilibrium and let $K_m(f) = \max_{k\in\mathcal{K}_m} N_k^h(f)$. We will consider three cases based on the value of $K_m(f)$ and show that under the assumptions of the proposition, any other equilibrium, $e \in \mathcal{E}_m$ has the same characterization.

Case 1: $K_m(f) = 0$. In this case there is no agent in S_m^w which connects to a hotspot. If there are no agents within any of the hotspots' service zones, then it is nothing to prove, since no agents make any choices. Otherwise, considering the equilibrium conditions for agents that fall within some hotspot we have:

$$U^w(M_m^w) > U^h(1). \tag{1.A.8}$$

It follows that no other equilibrium configuration can exist. Indeed, if $e \neq f$ is some other equilibrium configuration, we must have $N_l^h(e) \neq N_l^h(f)$, and thus $N_l^h(e) \geq 1$ yielding $N_m^w(e) \leq M_m^w - 1$. By Assumption 1.2 on utilities, we obtain:

$$U^w\left(M_m^w\right) \leq U^w\left(N_m^w(e)+1\right) \text{ and } U^h\left(N_l^h(e)\right) \leq U^h(1). \tag{1.A.9}$$

Since e is an equilibrium, we should have:

$$U^w\left(N_m^w(e)+1\right) \leq U^h\left(N_l^h(e)\right), \tag{1.A.10}$$

since no agent in S_l^h wishes to switch to WAN AP w_m. Combining inequalities (1.A.9) and (1.A.10) we obtain:

$$U^w\left(M_m^w\right) \leq U^h(1),$$

which contradicts inequality (1.A.8).

Case 2: $0 < K_m(f) = \max_{k \in \mathcal{K}_m} M_k^h$. In this case we have that there are no agents in C_m connected to the WAN AP w_m in configuration e and thus we have $\mathbf{N}_m(f) = \{M_k^h | k \in \mathcal{K}_m\}$. This can only be feasible if:

$$U^w\left(M_{C_m}^{\underline{w}}\right) \leq U^h(M_k^h),$$

for $k \in \mathcal{K}_m$. Using this inequality instead of (1.A.8) and following the steps similar to the Case 1 one can prove that no equilibrium e exists, such that $N_k^h(e) < M_k^h$ for some $k \in \mathcal{K}_m$.

Case 3: $0 < K_m(f) < \max_{k \in \mathcal{K}_m} M_k^h$. Consider any other equilibrium $e \neq f$ and note that $N_m^w(e) \geq M_{C_m}^{\underline{w}}$. Hence the inequalities (1.3) admit at most two integer solutions. It follows that, for some $K \geq 1$ we have that:

$$K - 1 \leq N_k^h(e) \leq K,$$

for $k \in \mathcal{K}_m$ such that $M_k^h \geq K$ and

$$N_k^h(f) = M_k^h,$$

otherwise. Hence e must be a fair equilibrium, characterized by the slicing plane at level $K_m(e) = K$. Since by part **(ii)** of Proposition 3.1, all fair equilibria are equivalent, we have, that $\mathbf{N}(e) \sim \mathbf{N}(f)$. □

4. Proof of Proposition 3.3

For any configuration $c \in \mathcal{T}_m$ we will refer to agents that have utility equal $U_m^{min}(c)$ as the "bottleneck" agents. Let $c \in \mathcal{T}_m$ be a configuration that maximizes utility of a bottleneck agent and $c \notin \mathcal{F}_m$. We will show that $U_m^{min}(c) \leq U_m^{min}(f)$, for all $f \in \mathcal{K}_m$. Since, by assumption of the proposition, all fair equilibria in S_m^w are equivalent, we have that $N_m^w(f) = N_m^w(f')$, for all $f, f' \in \mathcal{F}_m$. Thus to prove the proposition it suffices to consider the following three cases.

Case 1: $N_m^w(c) > N_m^w(f)$**, for all** $f \in \mathcal{F}_m$. In this case we have that $N_l^h(c) \leq N_l^h(f) - 1$ for at least one $l \in \mathcal{K}_m$. First we prove, that without loss of generality, one can assume that the bottleneck agents for configuration c are connected to a hotspot. Indeed, we have:

$$U^h\left(N_l^h(c)+1\right) \geq U^h\left(N_l^h(f)\right),$$

and

$$U^w\left(N_m^w(c)\right) \leq U^w\left(N_m^w(f)+1\right),$$

by Assumption 1.2 on utilities. Since in equilibrium f we must have $U^h(N_l^h(f)) \geq U^w(N_m^w(f)+1)$ we arrive at:

$$U^h\left(N_l^h(c)+1\right) \geq U^h\left(N_l^h(f)\right) \geq U^w(N_m^w(f)+1) \geq U^w\left(N_m^w(c)\right).$$

Hence, $U^h\left(N_l^h(c)+1\right) \geq U^w\left(N_m^w(c)\right)$ and thus the utility of the bottleneck agent stays the same or improves when an agent is switched from the WAN AP w_m to hotspot h_l.

Thus if c is maximizing the bottleneck among all configurations of agents choices, the bottleneck agents could be assumed to be connected to a hotspot. However, consider $l = \arg\max_{k\in\mathcal{K}_m} N_k^h(c)$. Then any agent connected to the hotspot h_l is the bottleneck for configuration c. Thus, since no agent connected to the WAN is the bottleneck for c, we have $U^h(N_l^h(c)) < U^w(N_m^w(c))$. Then we have the following chain of inequalities:

$$U^h(N_l^h(c)) \quad < \quad U^w(N_m^w(c)) \quad \underset{(a)}{\leq} \quad U^w(N_m^w(f)+1) \quad \underset{(b)}{\leq} \quad U^h(N_l^h(f)),$$

where inequality (a) follows from the assumption of Case 1, and inequality (b) – from the fact that f is an equilibrium. Thus $U^h(N_l^h(c)) < U^h(N_l^h(f))$ which means that $N_l^h(c) \geq N_l^h(f)+1$. Since f is a fair configuration, we have $\max_{k\in\mathcal{K}_m} N_k^h(f) \leq N_l^h(f)+1$. But then, $N_l^h(c) \geq \max_{k\in\mathcal{K}_m} N_k^h(f)$, and hence $U_m^{min}(c) \leq U_m^{min}(f)$.

Case 2: $N_m^w(c) < N_m^w(f)$, for all $f \in \mathcal{F}_m$. We first prove that no agents connected to the WAN can be the bottleneck for configuration c. Indeed, by assumption of this paragraph, we have that there exists at least one $l \in \mathcal{K}_m$ such that $N_l^h(c) \geq N_l^h(f)+1$. Now assume that the agents connected to the WAN are the bottleneck for configuration c, hence $U^w(N_m^w(c)) \leq U^h(N_k^h(c))$, for all $k \in \mathcal{K}_m$. Then we have the following chain of inequalities:

$$U^w(N_m^w(f)) \quad < \quad U^w(N_m^w(c)) \quad \leq \quad U^w(N_l^h(c)) \quad \leq \quad U^h(N_l^h(f)+1).$$

Hence $U^w(N_m^w(f)) < U^h(N_l^h(f)+1)$ which contradicts the fact that the agents connected to h_l in configuration f are in equilibrium. This shows that no agent connected to the WAN could be the bottleneck for the configuration c.

It follows that the agents within the hotspot h_n, such that $n = \arg\max_{k\in\mathcal{K}_m} N_k^h(c)$ are the bottleneck. Since there exists l such that $N_l^h(c) \geq N_l^h(f)+1$, we have that $N_n^h(c) \geq \max_{k\in\mathcal{K}_m} N_n^h(f)$, by the fair structure of f. This yields that $U_m^{min}(c) \leq U_m^{min}(f)$, which we claimed to show.

Case 3: $N_m^w(c) = N_m^w(f)$, for all $f \in \mathcal{F}_m$. First, we show again that no agent connected to the WAN could be the bottleneck for configuration c. Indeed, since $\mathbf{N}_m(f) \not\sim \mathbf{N}(c)$ we have that, by fair structure of f, there exists at least one $l \in \mathcal{K}_m$ such that $N_l^h(c) \geq N_l^h(f)+1$. Assuming that the agents connected to the WAN are the bottleneck in configuration c, we have the following chain:

$$U^w(N_m^w(f)) = U^w(N_m^w(c)) \leq U^h(N_l^h(c)) \leq U^h(N_l^h(f)+1).$$

Thus, $U^w(N_m^w(f)) \leq U^h(N_l^h(f)+1)$ indicating that f could not be an equilibrium configuration. This contradiction shows that the bottleneck agents for configuration c must be connected to hotspots. It is easy to see that $\max_{k\in\mathcal{K}_m} N_k^h(c) \geq \max_{k\in\mathcal{K}_m} N_k^h(f)$ which yields $U_m^{min}(c) \leq U_m^{min}(f)$.

5. Proof of Proposition 4.1

Prior to giving a proof of Proposition 4.1 we provide several technical lemmas.

<u>Lemma</u> **5.1.** *For any realization of the Poisson processes Π^a and Π^h consider a service zone associated with the WAN AP $w_0^\alpha \in \pi^{w,\alpha}$. Let*

$$L_k(K) = (M_k^h - K)\mathbf{1}_{\{M_k^h \geq K\}}, \quad P_k(K) = (M_k^h - K + 1)\mathbf{1}_{\{M_k^h \geq K\}}. \tag{1.A.11}$$

For any $m \in \mathbb{N}$ we have the following a.s. limits:

$$\lim_{\alpha\to\infty} \frac{H_0^\alpha}{\alpha} = \lambda^h\,, \quad \lim_{\alpha\to\infty} \frac{M_0^{w,\alpha}}{\alpha} = \lambda^a\,, \tag{1.A.12}$$

$$\lim_{\alpha\to\infty} \frac{\sum_{k\in\mathcal{K}_0^\alpha} L_k(K)}{H_0^\alpha} = \mathbb{E}_0^h\left[L_0(K)\right]\,, \quad \lim_{\alpha\to\infty} \frac{\sum_{k\in\mathcal{K}_0^\alpha} P_k(K)}{H_0^\alpha} = \mathbb{E}_0^h\left[P_0(K)\right]\,, \tag{1.A.13}$$

$$\lim_{\alpha\to\infty} \frac{M_{C_0}^{w,\alpha}}{\alpha} = \lambda^a(1-e^{-\lambda^h\pi|d|^2})\,, \qquad \lim_{\alpha\to\infty} \frac{M_{\bar{C}_0}^{w,\alpha}}{\alpha} = \lambda^a e^{-\lambda^h\pi|d|^2}\,, \tag{1.A.14}$$

$$\lim_{\alpha\to\infty} \mathbb{P}\left(\exists h_k \in S_0^{w,\alpha} : M_k^h \geq K\right) = 1\,,\ \forall K \geq 0\,. \tag{1.A.15}$$

Proof. The limits (1.A.12) follow by ergodicity [5] of the process π^a and π^h. One needs only to note that the ratio $\alpha/|S_0^{w,\alpha}|$ converges to 1 as $\alpha \to \infty$ since d (the radius of hotspot coverage) is bounded.

Consider now the limits (1.A.13). Note that for each k, and any fixed K, both $L_k(K)$ and $P_k(K)$ are functionals of the realization of processes Π^h and Π^a within some a.s. bounded region (Voronoi "flower" [12] associated with the Voronoi cell V_k^h). Thus $L_k(K)$ and $P_k(K)$ are "local statistics" as defined in [12], and thus one can use Theorem 3.1 therein to obtain these limits.

Now consider the limits (1.A.14). By (1.A.12) and (1.A.13) and noting that:

$$\sum_{k\in\mathcal{K}_0^\alpha} M_k^h = \sum_{k\in\mathcal{K}_0^\alpha} L_k(K)|_{K=0}\,,$$

we have:

$$\lim_{\alpha\to\infty} \frac{\sum_{k\in\mathcal{K}_0^\alpha} M_k^h}{\alpha} = \lambda^h \mathbb{E}_0^h\left[M_0^h\right]\,.$$

Evaluating this expectation, we get:

$$\mathbb{E}_0^h\left[M_0^h\right] = \mathbb{E}_0^h\left[\sum_{a_i\in\Pi^a(V_0^h)} \mathbf{1}_{\left\{a_i\in V_0^h\right\}} \mathbf{1}_{\{|a_i|\leq d\}}\right] = \mathbb{E}_0^h\left[\sum_{a_i\in\Pi^a} \mathbf{1}_{\{\Pi^h(B(a_i,|a_i|))=\emptyset\}} \mathbf{1}_{\{|a_i|\leq d\}}\right]\,,$$

where the second equality uses the fact that if $a_i \in V_0^h$ then there can be no other point of Π^h within the ball of radius $|a_i|$ centered at a_i. Now by independence of Π^h and Π^a and also using Campbell's formula and Slyvnyak's theorem (see e.g. [13]) we get:

$$\begin{aligned}\mathbb{E}_0^h\left[M_0^h\right] &= \mathbb{E}_0^h\left[\int_{x\in B(0,d)} \mathbf{1}_{\{\Pi^h(B(x,|x|))=\emptyset\}} \lambda^a dx\right] = \int_{x\in B(0,d)} e^{-\lambda^h\pi|x|^2}\lambda^a dx \\ &= \frac{\lambda^a}{\lambda^h}(1-e^{-\lambda^h\pi|d|^2})\,,\end{aligned}$$

from which the first limit in (1.A.14) follows. The second limit in (1.A.14) follows by taking into account the limit (1.A.13) and the first limit in (1.A.14).

Finally, to obtain the limit (1.A.15), we apply the Strong Law of Large Numbers to the sum of random variables $Z_k \triangleq \mathbf{1}_{\left\{M_k^h>K\right\}}$ to obtain:

$$\lim_{\alpha\to\infty} \frac{1}{H_0^\alpha} \sum_{k\in\mathcal{K}_0^\alpha} \mathbf{1}_{\left\{M_k^h>K\right\}} = \lim_{\alpha\to\infty} \frac{1}{H_0^\alpha} \sum_{k\in\mathcal{K}_0^\alpha} Z_k = \mathbb{P}(M_k^h > K) > 0 \quad a.s.\,. \tag{1.A.16}$$

Here we used the fact that the variables Z_k are i.i.d., since they depend on the number of points of homogeneous Poisson process sampled on disjoint sets S_k^h. Thus, at least one term in the sum in (1.A.16) is nonzero, for sufficiently large α, which proves the limit (1.A.15). □

Lemma 5.2. *Let Δ_i^α where $i = 1,2,3,4$ be defined as follows:*

$$\Delta_1^\alpha = M_0^{w,\alpha}, \quad \Delta_2^\alpha(K) = \sum_{k\in\mathcal{K}_0^\alpha} L_k(K), \quad \Delta_3^\alpha(K) = \sum_{k\in\mathcal{K}_0^\alpha} P_k(K), \quad \Delta_4^\alpha = M_{\bar{C}_0}^{w,\alpha}.$$

Then for each i, $1 \le i \le 4$ and any $C > 0$ we have:

$$\lim_{\alpha\to\infty} \mathbb{P}\left[|\Delta_i^\alpha - \mathbb{E}[\Delta_i^\alpha]| > C\sqrt{\alpha\log\alpha}\right] = 0. \tag{1.A.17}$$

Proof. To prove the lemma we will use Chebyshev's inequality:

$$\mathbb{P}\left[|\Delta_i^\alpha - \mathbb{E}[\Delta_i^\alpha]| > C\sqrt{\alpha\log\alpha}\right] \le \frac{\mathbf{var}\,[\Delta_i^\alpha]}{C^2\,\alpha\log\alpha}.$$

First we show that for $1 \le i \le 4$:

$$\mathbf{var}\,[\Delta_i^\alpha] = O(\alpha). \tag{1.A.18}$$

Indeed, $\Delta_1^\alpha = M_0^{w,\alpha}$ is just a Poisson random variable with average that scales linearly in α. Hence (1.A.18) is satisfied for $i = 1$. To obtain the bound on the variances of Δ_2^α and Δ_3^α we use Lemma 1 in [12], which yields:

$$\mathbf{var}\,[\Delta_2^\alpha(K)] = O(\alpha), \quad \mathbf{var}\,[\Delta_3^\alpha(K)] = O(\alpha).$$

Finally for the variance of $M_{\bar{C}_0}^{w,\alpha}$ observe that

$$M_{\bar{C}_0}^{w,\alpha} = M_0^{w,\alpha} - \Delta_2^\alpha(0).$$

Since the variances of both terms on the right are $O(\alpha)$ we get:

$$\mathbf{var}\,[\Delta_4^\alpha] = O(\alpha).$$

Now using Chebychev's inequality and (1.A.18) we obtain, for any $C > 0$,

$$\mathbb{P}\left(|\Delta_i^\alpha - \mathbb{E}[\Delta_i^\alpha]| > C\sqrt{\alpha\log\alpha}\right) = \frac{O(\alpha)}{\Theta(\alpha\log\alpha)} \to 0, \text{ when } \alpha\to\infty.$$

□

Lemma 5.3. *Under the scaling Assumption 4.1, the equilibrium f_0^α in $S_0^{w,\alpha}$ is unique and fair w.h.p..*

Proof. Using Lemma 5.2 we have that, eventually, $M_{\bar{C}_0}^{w,\alpha} \ge \bar{N}$ a.s. as $\alpha\to\infty$. Taking into account Assumption 4.1, the conditions of Proposition 3.2 hold w.h.p. Using Proposition 3.2 yields the statement of the lemma. □

Lemma 5.4. *For any equilibrium configuration f_0^α in $S_0^{w,\alpha}$ we have that:*

$$\max_{k\in\mathcal{K}_0^\alpha} N_k^h(f_0^\alpha) < \max_{k\in\mathcal{K}_0^\alpha} M_k^h, \text{ w.h.p.} \tag{1.A.19}$$

Proof. Note that (1.A.19) has a strict inequality. Thus (1.A.19) implies that the largest number of agents connected to any hotspot within S_0^w in equilibrium f_0^α is strictly less than the maximum number of agents in any one of the hotspots – at least asymptotically. We prove the lemma by contradiction. Suppose that there exists a sequence $\xi^\varepsilon = \{\alpha_n > 0 | \lim_{n\to\infty} \alpha_n = \infty\}$ with the following property. For any $\alpha \in \xi^\varepsilon$, f_0^α is such that for some $l^\alpha \in \mathcal{K}_0^\alpha$ we have $N_{l^\alpha}^h(f_0^\alpha) = \max_{k\in\mathcal{K}_0^\alpha} M_k^h$ with probability greater than ε. Then, for any $\alpha \in \xi^\varepsilon$:

$$J^\alpha(M_{l^\alpha}^h) \leq N_0^{w,\alpha}(f_0^\alpha)\,, \tag{1.A.20}$$

since no agent desires to switch to the WAN AP w_0 from the hotspot h_{l^α}. Now, note that f_0^α is fair w.h.p, by Lemma 5.3 and thus:

$$M_k^h - 1 \leq N_k^h(f_0^\alpha) \leq M_k^h\,,$$

where we took into account that there are no $k \in \mathcal{K}_0^\alpha$ such that $M_k^h > M_{l^\alpha}^h$. This yields, that at most one agent within each hotspot h_k, for $k \in \mathcal{K}_0^\alpha$ selects the WAN, thus

$$N_0^{w,\alpha}(f_0^\alpha) \leq M_{C_0}^{w,\alpha} + H^m\,, \tag{1.A.21}$$

Now, using Assumption 4.1 and Lemma 5.1, the inequalities (1.A.20) and (1.A.21) imply:

$$j\left(M_{l^\alpha}^h\right) \leq \lambda^a e^{-\lambda^h \pi d^2} + \lambda^h\,. \tag{1.A.22}$$

Taking into account that by Lemma 5.1 and Assumption 4.1:

$$\liminf_{\alpha\to\infty} \max_{k\in\mathcal{K}_0^\alpha} M_k^h = \infty\,, \qquad \lim_{N\to\infty} j(N) = \infty\,,\ a.s.$$

we find that the inequality (1.A.22) is violated with probability tending to 1 as $\alpha \to \infty$. Thus, ξ^ε can not exist for any $\varepsilon > 0$, which proves the lemma. $\square$

Lemma 5.5. *Consider a configuration f_0^α for service zone $S_0^{w,\alpha}$ and let $N_{max}^{h,\alpha}(f_0^\alpha) = \max_{k\in\mathcal{K}_m} N_k^h(f_0^\alpha)$. For any $\alpha > 0$, a necessary and sufficient condition for f_0^α to be an equilibrium w.h.p. is that f_0^α is a fair configuration that obeys either of the following:*

$$N_{max}^{h,\alpha}(f_0^\alpha) = 0\,, \quad J^\alpha(1) > M_0^{w,\alpha}\,, \tag{1.A.23}$$

$$N_{max}^{h,\alpha}(f_0^\alpha) \geq 1\,, \quad J^\alpha\left(N_{max}^{h,\alpha}(f_0^\alpha)\right) - 1 \leq N_0^{w,\alpha}(f_0^\alpha) < J^\alpha\left(N_{max}^{h,\alpha}(f_0^\alpha) + 1\right) \tag{1.A.24}$$

where $N_k^h(f_0^\alpha) = N_{max}^{h,\alpha}(f_0^\alpha)$ for all $k \in \mathcal{K}_0^\alpha$, such that $M_k^h \geq N_{max}^{h,\alpha}(f_0^\alpha)$, or:

$$N_{max}^{h,\alpha}(f_0^\alpha) \geq 1\,, \quad J^\alpha\left(N_{max}^{h,\alpha}(f_0^\alpha)\right) - 1 \leq N_0^{w,\alpha}(f_0^\alpha) < J^\alpha\left(N_{max}^{h,\alpha}(f_0^\alpha)\right)\,, \tag{1.A.25}$$

where $\exists k,l \in \mathcal{K}_0^\alpha$, such that $M_k^h, M_l^h \geq N_{max}^{h,\alpha}(f_0^\alpha)$, and $M_k^h = N_{max}^{h,\alpha}(f_0^\alpha)$, $M_l^h = N_{max}^{h,\alpha}(f_0^\alpha) - 1$.

Proof. We already proved in Lemma 5.3 that all equilibria in $S_0^{w,\alpha}$ have the same fair characterizations w.h.p. In case $N_{max}^{h,\alpha}(f_0^\alpha) = 0$ there are no agents connected to any hotspots in S_0^w. The necessary and sufficient condition for that, as follows from the inequality (1.2), is given by (1.A.23).

Consider the case $N_{max}^{h,\alpha}(f_0^\alpha) \geq 1$. First assume that for all $k \in \mathcal{K}_0^\alpha$, such that $N_{max}^{h,\alpha}(f_0^\alpha)$ we have that $N_k^h(f_0^\alpha) = N_{max}^{h,\alpha}(f_0^\alpha)$. By Lemma 5.4 we have $N_{max}^{h,\alpha}(f_0^\alpha) < \max_{k\in\mathcal{K}_0} M_k^h$, and thus we can use the equilibrium conditions (1.1) to obtain (1.A.24).

Now assume, instead, that there exist such $k,l \in \mathcal{K}_0^\alpha$, so that $M_k^h, M_l^h \geq N_{max}^{h,\alpha}(f_0^\alpha)$, and $M_k^h = N_{max}^{h,\alpha}(f_0^\alpha)$, $M_l^h = N_{max}^{h,\alpha}(f_0^\alpha) - 1$. For the hotspots having $N_{max}^{h,\alpha}(f_0^\alpha) - 1$ agents connected to them in configuration f_0^α, via equilibrium conditions (1.1) we get:

$$J^\alpha\left(N_{max}^{h,\alpha}(f_0^\alpha) - 1\right) - 1 \leq N_0^{w,\alpha}(f_0^\alpha) < J^\alpha\left(N_{max}^{h,\alpha}(f_0^\alpha)\right), \tag{1.A.26}$$

while for the hotspots having $N_{max}^{h,\alpha}(f_0^\alpha)$ agents connected to them:

$$J^\alpha\left(N_{max}^{h,\alpha}(f_0^\alpha)\right) - 1 \leq N_0^{w,\alpha}(f_0^\alpha) < J^\alpha\left(N_{max}^{h,\alpha}(f_0^\alpha) + 1\right), \tag{1.A.27}$$

Now, using monotonicity of $J^\alpha(\cdot)$, by combining (1.A.26) and (1.A.27) we get (1.A.25). □

Proof of Theorem 4.1. Let f_0^α denote an equilibrium configuration in the service zone $S_0^{w,\alpha}$ of the WAN AP $w_0^\alpha \in \pi_\alpha^w$. By Lemma 5.3 such configurations have equivalent and fair characterizations w.h.p, which gives Part 1 of the theorem. Let $N_{max}^{h,\alpha} = \max_{k \in \mathcal{K}_0^\alpha} N_k^h(f)$ where $f \in \mathcal{F}_0^\alpha$ is any fair equilibrium configuration. In what follows we will consider two cases that depend on whether the density of agents λ^a is less than the value $j(1)$. Our goal is to show that the $\lim_{\alpha\to\infty} N_{max}^{h,\alpha}(f_0^\alpha)$ exists.

Case 1: $\lambda^a < j(1)$. We will show that $\lambda^a < j(1)$ if and only if:

$$\lim_{\alpha\to\infty} N_{max}^{h,\alpha} = 0\,.$$

Indeed, the "only if" part follows from the condition (1.A.23) by dividing both sides by α and taking limits as $\alpha \to \infty$. Now using the limit (1.A.12) we obtain that $N_{max}^{h,\alpha} = 0$ w.h.p. implies $\lambda^a < j(1)$.

Next we prove that if $\lambda^a < j(1)$ then $N_{max}^{h,\alpha} = 0$ w.h.p. Indeed, by Lemma 5.1 we know that:

$$M_0^{w,\alpha} = \lambda^a\alpha + \varepsilon(\alpha)\,,$$

where $|\varepsilon(\alpha)| = O\left(\sqrt{\alpha\log\alpha}\right)$. But then, for sufficiently large α we have:

$$M_0^{w,\alpha} < J^\alpha(1)\,,$$

which, by Lemma 5.5 implies $N_{max}^{h,\alpha} = 0$ w.h.p.

Case 2: $\lambda^a \geq j(1)$. We first prove that $N_{max}^{h,\alpha}$ has a limit once $\alpha \to \infty$. Consider any sequence $\xi := \{\alpha_n | n \in \mathbb{N}\}$, where $\lim_{n\to\infty} \alpha_n = \infty$. We define the following disjoint subsequences of ξ:

$$\xi_1 = \left\{\alpha |\ \alpha \in \xi,\ 1 \leq N_{max}^{h,\alpha} < \max_{k\in\mathcal{K}_0^\alpha} M_k^h \text{ and } \forall k \in \mathcal{K}_0^\alpha,\ \text{s.t. } M_k^h \geq N_{max}^{h,\alpha},\ N_k^h(f^\alpha) = N_{max}^{h,\alpha}\right\}$$

$$\xi_2 = \left\{\alpha |\ \alpha \in \xi,\ 1 \leq N_{max}^{h,\alpha} < \max_{k\in\mathcal{K}_0^\alpha} M_k^h \text{ , and } \right.$$
$$\left. \exists k,l \in \mathcal{K}_0^\alpha,\ \text{s.t. } M_k^h, M_l^h \geq N_{max}^{h,\alpha},\ \text{and } N_k^h(f^\alpha) = N_{max}^{h,\alpha},\ N_l^h(f^\alpha) = N_{max}^{h,\alpha} - 1\right\}$$

$$\xi_3 = \left\{\alpha |\ \alpha \in \xi,\ 1 \leq N_{max}^{h,\alpha} = \max_{k\in\mathcal{K}_0^\alpha} M_k^h\right\}$$

$$\xi_4 = \left\{ \alpha |\ \alpha \in \xi,\ N^{h,\alpha}_{max} = 0 \right\}$$

Clearly, $\xi = \bigcup_{i=1}^{4} \xi_i$. However, by Lemma 5.4, the sequence ξ_3 is finite. Moreover, we have proved above that when $\lambda^a \geq j(1)$ the sequence ξ_4 is finite too. Thus, asymptotically, ξ consists only of the members of the sequences ξ_1 and ξ_2. Note that either ξ_1 or ξ_2 or both ξ_1 and ξ_2 have to be infinite, since ξ is infinite.

By definition of ξ_1 and ξ_2, we have, that $\max_{k\in\mathcal{K}_0^\alpha} M_k^h > N^{h,\alpha}_{max}$ when $\alpha \in \xi_1 \cup \xi_2$. Thus, if any of ξ_1 or ξ_2 is finite, to prove the statement of the theorem we have to show that $N^{h,\alpha}_{max}$ converges along the other infinite sequence. If both ξ_1 and ξ_2 are infinite, then we need to show that $N^{h,\alpha}_{max}$ is asymptotically the same along each subsequence, and in addition that:

$$\lim_{\alpha\in\xi_1,\ \alpha\to\infty} N^{h,\alpha}_{max} = \lim_{\alpha\in\xi_2,\ \alpha\to\infty} N^{h,\alpha}_{max}.$$

We will consider first the sequence ξ_1 and assume that it is infinite. We will show that:

$$\lim_{\alpha\in\xi_1,\ \alpha\to\infty} N^{h,\alpha}_{max} = K_1\,, \tag{1.A.28}$$

where K_1 is independent of α. We argue by contradiction. In particular, assume that there exist arbitrary large $\gamma,\delta \in \xi_1$ such that $N^{h,\gamma}_{max} \neq N^{h,\delta}_{max}$. Without loss of generality let $\gamma < \delta$, and consider the equilibrium conditions in $S_0^{w,\delta}$. By Lemma 5.5 we have that:

$$J^\delta\left(N^{h,\delta}_{max}\right) - 1 \leq N_0^w(f_0^\delta) < J^\delta\left(N^{h,\delta}_{max}+1\right),$$

Now multiplying these inequalities by γ/δ, and using Assumption 4.1, we obtain:

$$J^\gamma\left(N^{h,\delta}_{max}\right) - \gamma/\delta \leq \gamma/\delta N_0^{w,\delta}(f_0^\delta) < J^\gamma\left(N^{h,\delta}_{max}+1\right).$$

Note that by Lemma 5.1 we have:

$$\gamma/\delta N_0^{w,\delta}(f_0^\delta) = \gamma\left(\lambda^a e^{-\lambda^h\pi d^2} + \lambda^h \mathbb{E}_0^h\left[P_0^\delta(N^{h,\delta}_{max})\right]\right) + \varepsilon_1(\gamma,\delta)\,,$$

where, by Lemma 5.2, $|\varepsilon_1(\gamma,\delta)| = O\left(\gamma\sqrt{\log\delta/\delta}\right) = O\left(\sqrt{\gamma\log\gamma}\right)$. This yields:

$$J^\gamma\left(N^{h,\delta}_{max}\right) - 1 \leq \gamma\left(\lambda^a e^{-\lambda^h\pi d^2} + \lambda^h \mathbb{E}_0^h\left[P_0^\delta(N^{h,\delta}_{max})\right]\right) + \varepsilon_1(\gamma,\delta) < J^\gamma\left(N^{h,\delta}_{max}+1\right). \tag{1.A.29}$$

Now consider a fair configuration $\tilde{f}_0^\gamma$ for service zone $S_0^{w,\gamma}$, such that $\max_{k\in\mathcal{K}_0^\alpha} N_k^h(\tilde{f}_0^\gamma) = N^{h,\delta}_{max}$ and such that for all $k \in \mathcal{K}_0^\gamma$ for which $M_k^h \geq N^{h,\gamma}_{max}$ we have $N_k^h(\tilde{f}_0^\gamma) = N^{h,\delta}_{max}$. Clearly, in this case Lemma 5.1 and Lemma 5.2 yield:

$$N_0^w(\tilde{f}_0^\gamma) = \gamma\left(\lambda^a e^{-\lambda^h\pi d^2} + \lambda^h \mathbb{E}_0^h P_0^\delta\left(N^{h,\delta}_{max}\right)\right) + \varepsilon_2(\gamma)\,,$$

where $|\varepsilon_2(\gamma)| = O\left(\sqrt{\gamma\log\gamma}\right)$. By Assumption 4.1 (item 4) we have, for all integer K:

$$\lambda^a e^{-\lambda^h\pi d^2} + \lambda^h \mathbb{E}_0^h P_0^\delta(K) \neq j(K)\,,$$

which then translates the inequalities (1.A.29) into:

$$j\left(N^{h,\delta}_{max}\right) < \lambda^a e^{-\lambda^h\pi d^2} + \lambda^h \mathbb{E}_0^h P_0^\delta\left(N^{h,\delta}_{max}\right) < j\left(N^{h,\delta}_{max}+1\right). \tag{1.A.30}$$

Now note that $|\varepsilon_1(\gamma,\delta)+\varepsilon_2(\gamma)| = O\left(\sqrt{\gamma\log\gamma}\right) = o(\gamma)$. Hence, using (1.A.29), one gets

$$J^{\gamma}\left(N_{max}^{h,\delta}\right) - \gamma/\delta \le N_0^{w,\gamma}(\tilde{f}_0^{\gamma}) < J^{\gamma}\left(N_{max}^{h,\delta}+1\right),$$

once $\gamma < \delta$ are selected large enough. By Lemma 5.5 we have that $\tilde{f}_0^{\gamma}$ is a fair equilibrium that is different from f_0^{γ}. By Lemma 5.3 this can not happen w.h.p. Thus we obtain that

$$\lim_{\alpha\in\xi_1,\alpha\to\infty} N_{max}^{h,\delta} = K_1$$

for some positive integer K_1.

Now, if ξ_2 is finite, we are done, since asymptotically ξ consists only of ξ_1 and we have already shown that along ξ_1 the value of $N_{max}^{h,\alpha}$ has a limit. Now we will prove that if ξ_2 is infinite, then the value of $N_{max}^{h,\alpha}$ along ξ_2 also converges to a limit. Take any $\gamma\in\xi_2$ then, by Lemma 5.5 we have, that

$$J^{\alpha}(N_{max}^{h,\gamma}) - 1 \le N_0^{w,\gamma}(f^{\gamma}) < J^{\alpha}(N_{max}^{h,\gamma}).$$

Dividing these inequalities by γ, by Assumption 4.1, we have:

$$j(N_{max}^{h,\gamma}) - 1/\gamma \le \frac{N_0^{w}(f^{\gamma})}{\gamma} < j(N_{max}^{h,\gamma}). \tag{1.A.31}$$

We now show that $N_{max}^{h,\gamma} < K_0$ for some K_0 independent of γ. Indeed, otherwise, there exists a subsequence $\xi_5\subset\xi_2$, with $\lim_{\gamma\to\infty,\gamma\in\xi_5} N_{max}^{h,\gamma} = \infty$. Now using Lemma 5.1, we have that

$$\limsup_{\gamma\to\infty,\gamma\in\xi_5} \frac{N_0^{w}(f^{\gamma})}{\gamma} < \lambda^a e^{-\lambda^h\pi d^2} + \lambda^h \lim_{\gamma\to\infty,\gamma\in\xi_5} \mathbb{E}_0^h\left[P_0^{\gamma}\left(N_{max}^{h,\gamma}\right)\right] = \lambda^a e^{-\lambda^h\pi d^2}.$$

At the same time we have that

$$\lim_{\gamma\to\infty,\gamma\in\xi_5} j(N_{max}^{h,\gamma}) = \infty,$$

which means that the inequalities (1.A.31) could not be satisfied along the subsequence ξ_5. Thus we have a contradiction, and $\exists K_0$, such that $N_{max}^{h,\gamma} < K_0$.

Thus the set $\{N_{max}^{h,\gamma}|\gamma\in\xi_2\}$ is finite, hence if ξ_2 is infinite, at least some values from this set must realize infinitely often along ξ_2. Consider any value K which is achieved infinitely often along ξ_2, i.e. there exists a subsequence $\xi_6\subset\xi_2$, with $\sup\{\gamma|\gamma\in\xi_6\}=\infty$ and for any $\gamma\in\xi_6$, $N_{max}^{h,\gamma} = K$. Note that $N_0^{w,\gamma}(f_0^{\gamma})$ must satisfy:

$$M_{C_0}^{w,\gamma} + \sum_{k\in\mathcal{K}_0^{\gamma}} (M_k^h - K)\mathbf{1}_{\left\{M_k^h\ge K\right\}} < N_0^{w}(f_0^{\gamma}) < M_{C_0}^{w,\gamma} + \sum_{k\in\mathcal{K}_0^{\gamma}} (M_k^h - K + 1)\mathbf{1}_{\left\{M_k^h\ge K\right\}} \ w.h.p.,$$

since f_0^{γ} is asymptotically fair w.h.p. Dividing these inequalities by γ, and comparing to inequalities (1.A.31), one finds that K must satisfy:

$$l(K) \triangleq \lambda^a e^{\lambda^h\pi d^2} + \lambda^h\mathbb{E}_0^h[L_0(K)] < j(K) \le \lambda^a e^{\lambda^h\pi d^2} + \lambda^h\mathbb{E}_0^h[P_0(K)] \triangleq u(K), \tag{1.A.32}$$

where we used Lemma 5.1. Note that $u(K) = l(K+1)$ and thus the intervals $(l(K),u(K)]$ are disjoint for different integer K. Moreover $\bigcup_{K=1}^{\infty}(l(K),u(K)] = (0,u(1)]$. Since $j(K)$ is increasing in K and $\lim_{K\to\infty} j(K) = \infty$, there exists exactly one integer solution to the inequalities (1.A.32), since we assumed

$$j(1) \le \lambda^a,$$

and $u(1) = \lambda^a$. But then the value of $N_{max}^{h,\gamma}$ is asymptotically unique w.h.p., when $\gamma \in \xi_2$ and $\gamma \to \infty$.

We are left to show that if both ξ_1 and ξ_2 are infinite, then the asymptotic values K_1 and K_2 along ξ_1 and ξ_2 respectively satisfy $K_1 = K_2$. Observe that the condition (1.A.30) implies for K_1:

$$j(K_1) < u(K_1) < j(K_1 + 1).$$

Now, since K_2 is a unique integer solution to (1.A.32) we obtain that $K_2 = K_1$. Since $N_{max}^{h,\alpha}(f_0^\alpha) = K_1$ w.h.p. when $\alpha \in \xi_1$ and $N_{max}^{h,\alpha}(f_0^\alpha) = K_2 = K_1$ w.h.p when $\alpha \in \xi_2$, we obtain Part 2 of the theorem. Lastly, Part 3 of the theorem follows from the above analysis.

Notes

1. A more general case with utilities dependent on congestion and distance from a serving AP is treated in [11].
2. Note that we use letter M with different sub- and super- scripts to refer to the actual number of agents that fall within different sets, while we use the letter N to refer to the number of agents within different sets to refer to the agents *actually connected* to particular APs.
3. The ambiguity arises in the case when for a particular fair configuration $f \in \mathcal{F}_m$ we have $0 < N_k^h = K_m < \max_k M_k^h$, for all $k \in \mathcal{K}_m$ and for some $K_m \geq 0$. Then the upper surface of the "slicing plane", associated with this configuration can be drawn at either the levels K_m or $K_m + 1$.
4. Since, as we alluded above, the WAN service might be degrading slower with the number of connections than that of hotspots, the assumption that (1.5) holds may be reasonable.
5. This is, perhaps, not a bad assumption since WAN network would be carefully designed and optimized
6. See e.g. [9] for a nice comparison of WiFi vs. 3G technologies.

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